

Logical Types for Scheme

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What do these languages have in common?

- COBOL
- Scheme
- Ruby
- Haskell

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- COBOL [Komondoor 05]
- Scheme [Tobin-Hochstadt 06]
- Ruby [Furr 09]
- Haskell [Vytiniotis 10]

New static checks

What do these languages have in common?

- COBOL [Komondoor 05]
- Scheme [Tobin-Hochstadt 06]
- Ruby [Furr 09]
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Millions of lines of code



Types for Existing Languages

What's Hard?



How programmers reason

What's Hard?

Simple Types

How programmers reason

What's Hard?

Simple Types

Reflection

How programmers reason

What's Hard?

Simple Types

Parametricity

Reflection

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Simple Types

Dependent Types

Parametricity

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Simple Types

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Reflection

Generic Functions

How programmers reason

What's Hard?

Simple Types

Classes and Objects

Dependent Types

Parametricity

Reflection

Generic Functions

How programmers reason

Checking Existing Code

- New static checking is valuable for existing code
Maintenance, Optimization, Trust
- Work with existing idioms
Survey, Analyze, Design, Validate

What Can We Learn?

- New points in the design space

Ruby, Scheme, ...

- New type system ideas

Occurrence Typing



Occurrence Typing

Simple Examples

```
#lang typed/scheme

(: twice : Any -> Number)
(define (twice x)
  (if (number? x)
      (* 2 x)
      0))
```


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(: twice : Any -> Number)
(define (twice x)
  (if (number? x)
      (* 2 x)
      0))
```

Number_x

Simple Examples

```
#lang typed/scheme

(: twice : (U Number String) -> Number)
(define (twice x)
  (if (number? x)
      (* 2 x)
      (* 2 (string-length x))))
```

Simple Examples

```
#lang typed/scheme

(: twice : (U Number String) -> Number)
(define (twice x)
  (if (number? x)
      (* 2 x)
      (* 2 (string-length x))))
```

Number_x

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(or (number? x) (string? x))
         (twice x)]

        [else 0]))
```

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(or (number? x) (string? x))
         (twice x)]

        [else 0]))
```

$\text{Number}_x \vee \text{String}_x$

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(or (number? x) (string? x))
         (twice x)]

        [else 0]))
```

$\text{Number}_x \vee \text{String}_x \quad (\text{U Number String})_x$

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(and (number? x) (string? y))
         (+ (twice x) (twice y))]

        [else 0]))
```

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(and (number? x) (string? y))
         (+ (twice x) (twice y))]
        [else 0]))
```

$\text{Number}_x \wedge \text{String}_y$

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(and (number? x) (string? y))
         (+ (twice x) (twice y))]
        [(number? x) (* 2 y)]
        [else 0]))
```

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(and (number? x) (string? y))
         (+ (twice x) (twice y))]
        [(number? x) (* 2 y)]
        [else 0]))
```

$\overline{\text{Number}_x} \vee \overline{\text{String}_y}$

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(and (number? x) (string? y))
         (+ (twice x) (twice y))]
        [(number? x) (* 2 y)]
        [else 0]))
```

$\overline{\text{Number}_x} \vee \overline{\text{String}_y}$, Number_x $\overline{\text{String}_y}$

Logical Combination

```
#lang typed/scheme

(: twice : (U Number String) -> Number)

(: g : Any (U Number String) -> Number)
(define (g x y)
  (cond [(and (number? x) (string? y))
         (+ (twice x) (twice y))]
        [(number? x) (* 2 y)]
        [else 0]))
```

$(U \text{ Number String})_y$, $\overline{\text{String}}_y$ Number_y

Data Structures

```
#lang typed/scheme

(: twice-car : (Pair Any Any) -> Number)
(define (twice-car x)
  (if (number? (car x))
      (* 2 (car x))
      0))
```

Data Structures

```
#lang typed/scheme

(: twice-car : (Pair Any Any) -> Number)
(define (twice-car x)
  (if (number? (car x))
      (* 2 (car x))
      0))
```

Number_car(x)

Abstraction

```
#lang typed/scheme
```

```
(: car-num? : (Pair Any Any) -> Boolean : Number @ car)  
(define (car-num? x)  
  (number? (car x)))
```

Abstraction

```
#lang typed/scheme
```

```
(: car-num? : (Pair Any Any) -> Boolean : Number @ car)  
(define (car-num? x)  
  (number? (car x)))
```




Propositional Logic

Judgments

$$\Gamma \quad e : \mathbf{T} \ ; \ \phi_1 \ | \ \phi_2$$

Judgments

$$\Gamma \quad e : T \ ; \ \phi_1 \ | \ \phi_2$$
$$e ::= n \ | \ c \ | \ (\lambda \ x : T . e) \ | \ (e \ e) \ | \ (\text{if } e \ e \ e)$$

Judgments

$$\Gamma \quad e : \mathbf{T} \ ; \ \phi_1 \ | \ \phi_2$$
$$\mathbf{T} ::= \mathbf{Number} \ | \ (\mathbf{U} \ \mathbf{T} \ \dots) \ | \ \#\mathbf{t} \ | \ \#\mathbf{f} \ | \ (\mathbf{x}:\mathbf{T} \ \rightarrow \ \mathbf{T} \ : \ \phi \ | \ \phi)$$

Judgments

$$\Gamma \quad e : \mathbf{T} \ ; \ \phi_1 \ | \ \phi_2$$
$$\phi ::= \mathbf{T}_\pi(\mathbf{x}) \ | \ \overline{\mathbf{T}_\pi(\mathbf{x})} \ | \ \phi_1 \vee \phi_2 \ | \ \phi_1 \wedge \phi_2 \ | \ \phi_1 \supset \phi_2$$

Judgments

$$\Gamma \quad e : \mathbf{T} \ ; \ \phi_1 \ | \ \phi_2$$
$$\phi ::= \mathbf{T}_\pi(\mathbf{x}) \ | \ \overline{\mathbf{T}_\pi(\mathbf{x})} \ | \ \phi_1 \vee \phi_2 \ | \ \phi_1 \wedge \phi_2 \ | \ \phi_1 \supset \phi_2$$

Judgments

$$\Gamma \quad e : \mathbf{T} \ ; \ \phi_1 \ | \ \phi_2$$
$$\Gamma ::= \mathbf{T}_\pi(\mathbf{x}) \ \dots$$

Judgments

$$\Gamma \quad e : \mathbf{T} \ ; \ \phi_1 \ | \ \phi_2$$
$$\Gamma ::= \phi \ \dots$$

Judgments

$\Gamma \quad \phi$

Judgments

$\Gamma \quad \phi$

$\frac{}{\text{Number}_x} \vee \frac{}{\text{String}_y}, \text{Number}_x \quad \frac{}{\text{String}_y}$

Typing

```
(if e1 e2 e3)
```

Typing

(if e_1 e_2 e_3)

$\Gamma \quad e_1 : T_1 ; \phi_{-+} \mid \phi_{-}$

Typing

(if e_1 e_2 e_3)

$\Gamma \quad e_1 : T_1 ; \phi_{-+} | \phi_{--}$

$\Gamma, \phi_{-+} \quad e_2 : T ; \phi_{1+} | \phi_{1-}$

$\Gamma, \phi_{--} \quad e_3 : T ; \phi_{2+} | \phi_{2-}$

Typing

`(if e1 e2 e3)`

$\Gamma \quad e_1 : T_1 ; \phi_{-+} \mid \phi_{--}$

$\Gamma, \phi_{-+} \quad e_2 : T ; \phi_{1-+} \mid \phi_{1--}$

$\Gamma, \phi_{--} \quad e_3 : T ; \phi_{2-+} \mid \phi_{2--}$

Typing

$$\Gamma \quad (\text{if } e_1 \ e_2 \ e_3) : \mathbf{T} \ ; \ \phi_{1_+} \vee \phi_{2_+} \mid \phi_{1_ -} \vee \phi_{2_ -}$$
$$\Gamma \quad e_1 : \mathbf{T}_1 \ ; \ \phi_{_+} \mid \phi_{_ -}$$
$$\Gamma, \phi_{_+} \quad e_2 : \mathbf{T} \ ; \ \phi_{1_+} \mid \phi_{1_ -}$$
$$\Gamma, \phi_{_ -} \quad e_3 : \mathbf{T} \ ; \ \phi_{2_+} \mid \phi_{2_ -}$$

Typing

$$\Gamma \quad (\text{if } e_1 \ e_2 \ e_3) : \mathbf{T} \ ; \ \phi_{1_+} \vee \phi_{2_+} \mid \phi_{1_ -} \vee \phi_{2_ -}$$
$$\Gamma \quad e_1 : \mathbf{T}_1 \ ; \ \phi_{_+} \mid \phi_{_ -}$$
$$\Gamma, \phi_{_+} \quad e_2 : \mathbf{T} \ ; \ \phi_{1_+} \mid \phi_{1_ -}$$
$$\Gamma, \phi_{_ -} \quad e_3 : \mathbf{T} \ ; \ \phi_{2_+} \mid \phi_{2_ -}$$

Typing

$$\Gamma \quad (\text{if } e_1 \ e_2 \ e_3) : \mathbf{T} \ ; \ \phi_{1_+} \vee \phi_{2_+} \mid \phi_{1_+} \vee \phi_{2_+}$$
$$\Gamma \quad e_1 : \mathbf{T}_1 \ ; \ \phi_{_+} \mid \phi_{_-}$$
$$\Gamma, \phi_{_+} \quad e_2 : \mathbf{T} \ ; \ \phi_{1_+} \mid \phi_{1_+}$$
$$\Gamma, \phi_{_-} \quad e_3 : \mathbf{T} \ ; \ \phi_{2_+} \mid \phi_{2_+}$$

Typing

$\Gamma \quad \mathbf{x} : \mathbf{T}$

Typing

$$\Gamma \quad \mathbf{x} : \mathbf{T}$$
$$\Gamma \quad \mathbf{T}_x$$

Try Typed Scheme

Installer and Documentation
<http://www.plt-scheme.org>

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